# $\varepsilon$-Distance via Lévy-Prokhorov Lifting

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## Abstract

My talk will focus on a particular distance, $\varepsilon$-distance for probabilistic processes, that is defined using a notion of behavioural equivalence, $\varepsilon$-bisimilarity. $\varepsilon$-distance is intuitively easy to understand, it relates systems that are conceptually close (for example, an imperfect implementation is close to its specification), and it comes equipped with a natural notion of $\varepsilon$-coupling. However, the distance has not been particularly used or studies since its introduction. One reason could be the lack of generic results.

The most studied and accepted pseudometric for probabilistic processes is one obtained by lifting the Kantorovich distance between distributions, which itself is a lifting of a base distance between states. It comes with many theoretical and motivating results, in particular it is the (greatest) fixpoint of a given functional and defines a functor on (complete) pseudometric spaces. It is also the foundation for a categorical lifting of pseudometrics.

In recent joint work with Josée Desharnais, we somewhat surprisingly discovered that $\varepsilon$-distance is also the greatest fixpoint of a functional and provides a functor. The latter is obtained by replacing the Kantorovich distance in the lifting functional with the Lévy-Prokhorov distance, which is another lifting to distributions of a base distance between states. In addition, $\varepsilon$-couplings and $\varepsilon$-bisimulations have a coalgebraic characterization, related to coalgebraic simulation and bisimulations.

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